Thermally induced flow of two immiscible stratified liquids in a porous body at a low Rayleigh number

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If nuclear waste is buried deep in the rock where the ground water is saline, the generated heat will force the saline water to move upward. If the generated heat is great enough, the saline water will rise through the fresh ground water, which is floating on top of the saline ground water. A theoretical estimate and a simple laboratory experiment with a combined Hele-Shaw and heat conduction cell show that the critical power, required for the interface between saline and fresh water to reach the free surface, is proportional to the relative density difference and to the thickness of the fresh water layer, whereas it is inversely proportional to the volume expansion coefficient of water. The result also shows that the critical power is independent of the distance between the depository and the interface.

Keywords: thermal convection; stratified flow; porous media

Introduction

If nuclear waste is buried deep in the rock, the generated heat will force the ground water to move. This kind of problem on thermal convection in saturated permeable bodies was first studied by Horton and Rogers (1945). Later Wooding treated the problem (1957, 1958). A comprehensive survey has been presented by Combarnous and Bories (1975). An analysis applied to nuclear waste deposition in rock has been presented by Hodgkinson (1980).

An interesting variant of this kind of thermally induced convection is concerned with stratified ground water where the stratification is caused by a step in salinity. If the depository is located below the stratification, a possible contamination of the saline water might be trapped there and prevented from reaching the fresh water and further to the biosphere. However, the power generated in the depository might be so great that the stratification is destabilized by the temperature gradient. In such a case the protecting stratification is spoiled.

Therefore, it is an urgent task to estimate under which circumstances the stratification is maintained. An analysis of the above problem is extremely difficult even if the complex structure of the rock is disregarded. Nevertheless, the fundamental character of the problem can be estimated and also studied experimentally in the laboratory. The basis for such experiments is the analogy between the flow through a porous body and through a narrow slot, called a Hele-Shaw cell. A new approach is to make one (or both) of the walls of the cell of a material that conducts heat. In this way it is possible to study thermally induced flow where heat is conducted as well as convected. Combarnous and Bories (1975) claim that a Hele-Shaw cell is less suited, but strictly speaking this is only true if the purpose is to study convection at infinite Rayleigh number. The Hele-Shaw cell also permits the simulation of fracture zones in a rock mass. This might be of interest from a practical point of view; a fracture zone, serving as a hydraulic short circuit, can affect the displacement of the interface between the saline and fresh water.

This kind of modified Hele-Shaw cell, as well as conventional ones, are well suited for experiments and particularly for visualizations. However, it can only be used to simulate two-dimensional (2-D) flow. This is a complication in this case, since a 2-D analysis would be very complicated and miss the elegance and simplicity of Hodgkinson's solution. As the goal of the present study is qualitative results achieved with simple means, the theory assumes three-dimensional (3-D) flow with axial symmetry and the experiments are 2-D.

From a practical point of view the dominant part of the heat is conducted through the rock mass and a lesser part is convected with the moving water. The analysis and experiment described below is confined to this case.

Theoretical background

Consider a homogeneous isotropic rock mass where the ground water consists of two strata; fresh water floating on top of salt water. The difference between the densities is so small that a heat source in the rock mass may force the salt water to reach the free surface (Figures 1a–c). The salt water and fresh water are considered immiscible, i.e., diffusion of salinity across the interface is neglected.

The problem contains two kinds of nonlinearities that make it very difficult to find a solution. The first one is that the heat transport equations for the two strata contain a convective term. The second one is that the free surface of the upper stratum and the interface between the two strata are unknown

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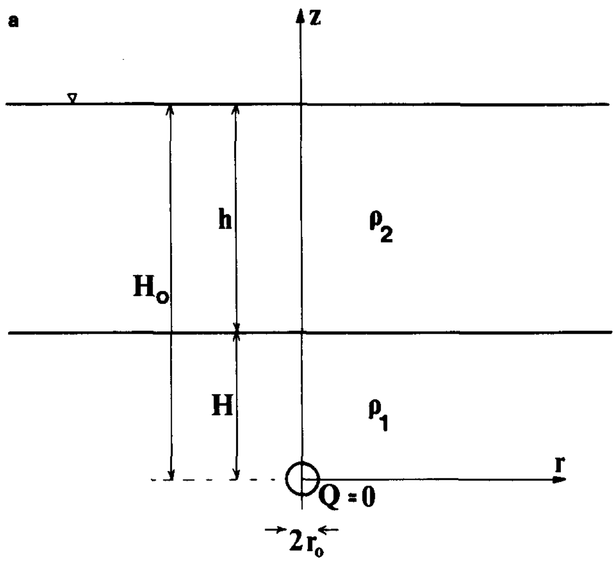

Fig. 1(a)

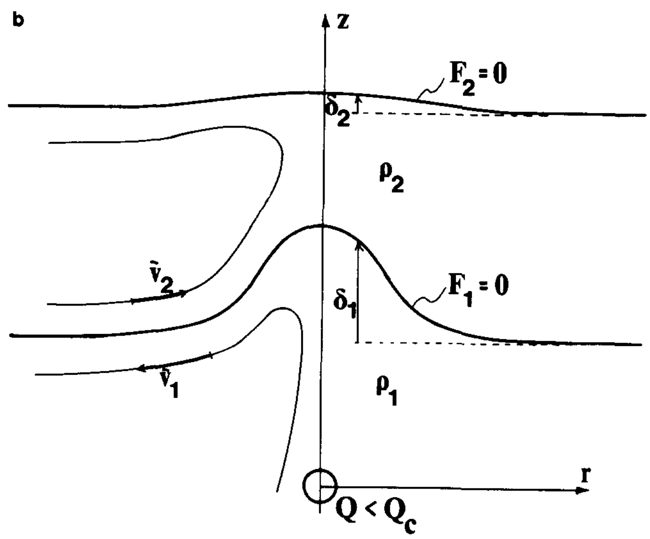

Fig. 1(b)

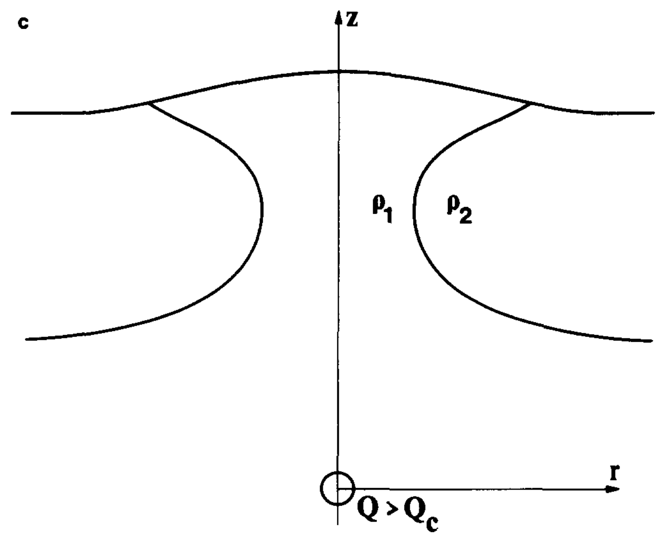

Fig. 1(c)

and a part of the problem. If the upper stratum is absent, the problem can be treated with some success. If the Rayleigh number is small and the material properties are constant, the problem can be linearized and solved. As mentioned above, Hodgkinson (1980) has given a solution to this case that becomes especially simple and appealing if the heat source, and thereby the temperature field, is spherical.

Let $\mathbf{r} = (x, y, z)$ be the space coordinates and t the time. The average heat conduction coefficient, density, and heat capacity of the rock and water together are denoted λ, ρ, and c. They are dominated by the properties of the rock. The densities and heat expansion coefficients of the two liquids are ρ_1 and ρ_2 and

Figure 1(a) The two liquids, numbers 1 and 2, in equilibrium when no heat is supplied at the source. Definitions of parameters
Figure 1(b) The two liquids, numbers 1 and 2, in motion when the supplied power is less than the critical power. Definition of variables
Figure 1(c) The two liquids, numbers 1 and 2, in motion when the supplied power is greater than the critical power

Notation

c	Heat capacity of the solid, $\mathrm{Jkg^{-1}K^{-1}}$
c_i	Heat capacity of a liquid, $\mathrm{Jkg^{-1}K^{-1}}$
C	Salt concentration in ground water
d	Width of the slot, m
F_i	Implicit function of a surface
g	Gravity acceleration, $\mathrm{ms^{-2}}$
G	Radial component of a stream function, $\mathrm{m^2K}$
h	Thickness of the upper stratum, m
H	Distance between the heat source and the interface, m
H_0	$H + h$, m
k	Permeability, $\mathrm{m^2}$
L	Size of the Hele Shaw cell, m
n	Porosity
p_i	Pressure of a liquid, Pa
Q	Power of the source, W
$\mathbf{r}$	Cylindrical coordinate (r, z), m
r	Radial coordinate, m
s	Auxiliary variable
t	Time
T	Temperature, K
T_∞	Undisturbed temperature, K
$\mathbf{u}$	Pore velocity of a liquid, $\mathrm{ms^{-1}}$
$\mathbf{v}$	Velocity of a liquid, $\mathrm{ms^{-1}}$
z	Vertical coordinate, m

Greek symbols

α	Compression coefficient of the solid, $\mathrm{Pa^{-1}}$
β	Compression coefficient of the liquid, $\mathrm{Pa^{-1}}$
γ_i	Thermal expansion coefficient of a liquid, $\mathrm{K^{-1}}$
δ_i	Vertical displacement of a surface, m
ϑ	Thermal expansion coefficient, $\mathrm{K^{-1}}$
λ	Heat conduction coefficient of the solid, $\mathrm{Wm^{-1}K^{-1}}$
μ	Viscosity, $\mathrm{kg^{-1}s^{-1}}$
ξ	Auxiliary variable, m
ρ	Density of the solid, $\mathrm{kg^{-3}}$
ρ_i	Density of a liquid, $\mathrm{kg^{-3}}$
χ	Coefficient relating viscosity and temperature, $\mathrm{K^{-1}}$

$i = 0$, undisturbed value; $i = 1$, lower liquid; $i = 2$, upper liquid

γ_1 and γ_2, respectively. The liquids have the same viscosity μ. The velocities (flux per unit area) and pressures of the liquids are $\mathbf{v}_1$ and $\mathbf{v}_2$ and p_1 and p_2 respectively. In the rock there is a heat source Q. The implicit equations of the free surface and the interface are denoted $F_2(x, y, z, t) = 0$ and $F_1(x, y, z, t) = 0$, respectively. The explicit equations are $z = \delta_2(x, y, t)$ and $z = \delta_1(x, y, t)$ (Figure 1b). The densities are supposed to obey Boussinesq's assumption, i.e., ρ_1 and ρ_2 are considered as constants except for the buoyancy influence.

The equations describing the heat and fluid flow are the energy conservation equations, the mass conservation equation, and Darcy's law in both strata. Besides this the free surface and the interface are stream surfaces. Then the system of equations reads

$$\frac{\partial \rho_1}{\partial t} + \nabla\cdot\left(\frac{\rho_1 \mathbf{v}_1}{n}\right) = 0 \tag{1}$$

$$\nabla p_1 + \frac{\mu}{k}\mathbf{v}_1 = -\rho_1 \mathbf{g} \tag{2}$$

$$\rho c\frac{\partial T}{\partial t} - \lambda\nabla^2 T + \rho_1 c_1 \mathbf{v}_1\cdot\nabla T = 0 \tag{3}$$

$$\frac{\partial \rho_2}{\partial t} + \nabla\cdot\left(\frac{\rho_2 \mathbf{v}_2}{n}\right) = 0 \tag{4}$$

$$\nabla p_2 + \frac{\mu}{k}\mathbf{v}_2 = -\rho_2 \mathbf{g} \tag{5}$$

$$\rho c\frac{\partial T}{\partial t} - \nabla^2 T + \rho_2 c_2 \mathbf{v}_2\cdot\nabla T = 0 \tag{6}$$

$$\frac{\partial F_1}{\partial t} - \frac{k}{\mu n}(\nabla F_1)\cdot(\nabla p_1 + \rho_1 \mathbf{g}) = 0 \tag{7}$$

$$\frac{\partial F_1}{\partial t} - \frac{k}{\mu n}(\nabla F_1)\cdot(\nabla p_2 + \rho_2 \mathbf{g}) = 0 \tag{8}$$

$$\frac{\partial F_2}{\partial t} - \frac{k}{\mu n}(\nabla F_2)\cdot(\nabla p_2 + \rho_2 \mathbf{g}) = 0 \tag{9}$$

$$1/\mu = (1/\mu_0)(1 - \chi(T - T_0)) \tag{10}$$

$$\rho_1 = \rho_{10}(1 + \beta_1(p - p_0))(1 + \gamma_1(T - T_\infty)) \tag{11}$$

$$\rho_2 = \rho_{20}(1 + \beta_2(p - p_0))(1 + \gamma_2(T - T_\infty)) \tag{12}$$

$$n = n_0(1 + \alpha(p - p_0))(1 + \vartheta(T - T_0)) \tag{13}$$

Boussinesq's approximation means that $\gamma_1 = \gamma_2 = 0$ in Equations 1, 3, 4, and 6.

Besides these equations there are boundary conditions (BC) and initial conditions (IC). Thus, if the rock is infinite and if the liquids occupy a half space, the temperature, velocities, and pressures are undisturbed far from the source, the power of the source is known, and the pressure is a continuous function. When $t = 0$ the state is known

$$(\mathrm{BC})\quad \begin{cases} T(\mathbf{r}\to\infty) = T_\infty & (14)\\ \displaystyle\int_\Omega \frac{\partial T}{\partial \hat{n}}\cdot ds = Q/\lambda & (15)\\ \mathbf{v}_1(\mathbf{r}_1\to\infty) = \mathbf{v}_2(\mathbf{r}\to\infty) = 0 & (16)\\ p_1(\mathbf{r}\to\infty) = -\rho_1 g z & (17)\\ p_2(\mathbf{r}\to\infty) = -\rho_2 g z & (18)\\ p_1(\mathbf{r}\in F_1) = p_2(\mathbf{r}\in F_1) & (19)\\ p_2(\mathbf{r}\in F_2) = 0 & (20)\end{cases}$$

$$(\mathrm{IC})\quad \begin{aligned} &T(t<0) = T_\infty\\ &\mathbf{v}(t<0) = \mathbf{v}_0\\ &p(t<0) = p_0\\ &F_1 : z = H\\ &F_2 : z = H_0\end{aligned} \tag{21}$$

The boundary condition 15 means that the temperature is constant in the depository in contrast to Hodgkinson's solution where Q is a heat source distribution in Equation 3. As mentioned previously, the above system contains two kinds of nonlinearities, namely the presence of term number three in the energy Equations 3 and 6 and the interfacial conditions in Equations 7 and 8. A third complication, which is often neglected, is that the viscosity of water varies strongly with the temperature.

Hodgkinson's solution is derived under the following assumptions:

$$\rho_1 = \rho_2 = \rho_0 \tag{22}$$

$$\gamma_1 = \gamma_2 = \gamma_0 \tag{23}$$

$$\chi = \alpha = \vartheta = \beta_1 = \beta_2 = 0 \tag{24}$$

$$F_2 \equiv z - H_0 = 0 \tag{25}$$

$$\Omega: x^2 + y^2 + z^2 = r_0^2 \tag{26}$$

$$\mathrm{Ra} = \frac{k\gamma_0 g \rho_0 c_0 Q}{4\pi\lambda^2\mu} \to 0 \tag{27}$$

where $H_0 = H + h$. The region Ω is the spherical heat source, in this case a cavity. Since the solution is cylindrically symmetrical it is presented in cylindrical coordinates (r, z), where $r = \sqrt{x^2 + y^2}$. According to Hodgkinson the displacement of the free surface $\delta(r, z = H)$ then reads

$$\delta(r) = \frac{2\gamma_0 H_0}{r^2 + H_0^2} G((r^2 + H_0^2)^{1/2}) \tag{28}$$

where the radial component of the stream function $G(s)$ reads

$$G(s) = \frac{1}{s}\int_0^s \xi^2 T(\xi)d\xi \tag{29}$$

In the simplest case, i.e., steady heat flow from a spherical cavity, the temperature becomes

$$T(\xi) = \frac{Q}{4\pi\lambda\xi} \tag{30}$$

where Q is the constant heat flux from the cavity. Then

$$\delta(r) = \frac{Q\gamma_0}{4\pi\lambda}\frac{1}{(1 + (r/H_0)^2)^{1/2}} \tag{31}$$

where $\delta(r)/H_0$ is supposed to be negligible. Thus, an estimate of the maximum displacement is

$$\delta(0) = \frac{Q\gamma_0}{4\pi\lambda} \tag{32}$$

The question is now whether the corresponding case can be solved for the two regions $-\infty < z < h$ and $h < z < H_0$. Unfortunately this is impossible. The reason is that δ_1 is not small since the relative density difference is very small. Thus, the boundary $F_2 = 0$ can be linearized but not $F_1 = 0$.

A simple estimate can be achieved if this problem is considered as an analogy with the problem in oil production in geological formations where the oil is floating on the ground water. If the production rate of oil is high, the water bed rises under the well. If the rate becomes too high, water will be sucked into the well. The rise height of the interface between oil and water is proportional to the density difference. This phenomenon, which is called water coning, is discussed in detail in section 8.10 of Muskat's textbook (1937).

In the present case the displacement of the interface is caused by the buoyancy instead of the suction of the well. Thus, it is assumed that the displacement of the interface is proportional to the density difference $\rho_{10} - \rho_{20}$ and to the equivalent suction, i.e., to the displacement δ_2, which in turn is given by Hodgkinson's solution Equation 31.

$$\delta_2 = \frac{Q\gamma_0}{4\pi\lambda} \tag{33}$$

$$\delta_1 = \frac{Q\gamma_0}{4\pi\lambda}\frac{\rho_{20}}{\rho_{10} - \rho_{20}} \tag{34}$$

where the condition $\delta_1 \ll H$ is violated. Then the following equation

$$\delta_1(0) = h \tag{35}$$

means that the heavy liquid has broken through the light one. Thus, the critical power reads

$$Q_c = \frac{4\pi\lambda(\rho_{10} - \rho_{20})h}{\gamma_{10}\rho_{10}} \tag{36}$$

This condition shows that the critical power Q_c is independent of H. Since $2r_0$ is a characteristic length of the problem, it is convenient to choose a reference value of the critical power Q_c', such that

$$Q_c' = \frac{4\pi\lambda(\rho_{10} - \rho_{20})2r_0}{\gamma_{10}\rho_{10}} \tag{37}$$

The definition in Equation 37, which in principle is arbitrary, implies that the dimensionless critical power is proportional to the dimensionless thickness of the upper stratum.

$$\hat{Q}_c \equiv Q_c/Q_c' = (h/2r_0) \tag{38}$$

The choice of Equations 37 and 38 fits the experiments below, since the only parameters that are varied are h and H.

When $\alpha = \vartheta = \beta_1 = \beta_2 = 0$ the flow is quasi-steady. A rough criterion that tells if quasi-steady flow is true is derived if the thermal and hydraulical diffusivities are compared. The thermal diffusivity κ_t reads

$$\kappa_t = \frac{\lambda}{\rho c} \tag{39}$$

and the hydraulical diffusivity reads

$$\kappa_1 = \frac{k}{n_0\mu(\alpha + \beta)} \tag{40}$$

Thus,

$$\frac{\lambda n_0\mu(\alpha + \beta)}{\rho c k_0} \ll 1 \tag{41}$$

is a simple criterion for quasi-steady flow, provided the Rayleigh number is negligible.

The above analysis assumes 3-D heat and fluid flow that is undisturbed at infinity. In the case of a laboratory experiment in a Hele-Shaw cell the flow is 2-D and confined. In contrast to the above analysis the Hele-Shaw cell contains its size as a characteristic length. Further, the thermally induced velocities in the two strata are counteracting at their interface. If the Reynolds number is very small this effect is probably negligible, but if the Reynolds number increases, something like Kelvin-Helmholz instability might occur. Likewise the surface tension at the free surface and at the interface in a Hele-Shaw cell is not a good picture of the corresponding phenomena in a porous body (and of course not to the phenomena in a fractured rock). Finally, the flow in a Hele-Shaw cell is quasi-steady since the hydraulic diffusivity is zero.

An interesting consequence of the simple 3-D theory summarized in Equations 36 and 41 is that the critical power is independent of the permeability. Thus, the presence of a zone with higher permeability, or a hydraulic short circuit, will have a small influence on the shape of the interface as well as on the critical power. If the Rayleigh number increases, such a zone will have a strong influence on both the shape of the interface and consequently on the critical power.

Experiment

The conclusions about the critical power that were drawn in the previous section are founded on a number of simplifications. Therefore, it is important to test the theoretical results with experiments. As mentioned in the introduction, this is done with a hybrid between a heat conduction cell and a Hele-Shaw cell. The objective is to measure the critical power as a function of the thickness h of the upper stratum and the distance H between the heat source and the interface.

The design of the experiment is complex and a description of the design principle and the experimental setup is presented below. The walls of the Hele-Shaw cell are replaceable. One of them is transparent, i.e., made of glass or preferably Plexiglas. The other one is made of metal or a nonconducting material if the study concerns high Rayleigh numbers. Somewhere in the cell there is a heat source. The heat is forced to propagate in the plane of the cell by a thick insulation on the back of the metal wall. If a slot is milled in the Plexiglas wall, this slot simulates a hydraulically conducting zone (Figure 2).

The gap between the walls of the Hele-Shaw cell is filled with two immiscible liquids whose densities differ very little. The heat source is a copper bar that goes through the metal plate and the slot. It is connected to an electric heater, in this case a soldering iron. This arrangement implies some thermal inertia of the heat source, which is a clear disadvantage if the aim is to study the transient development of a destabilization.

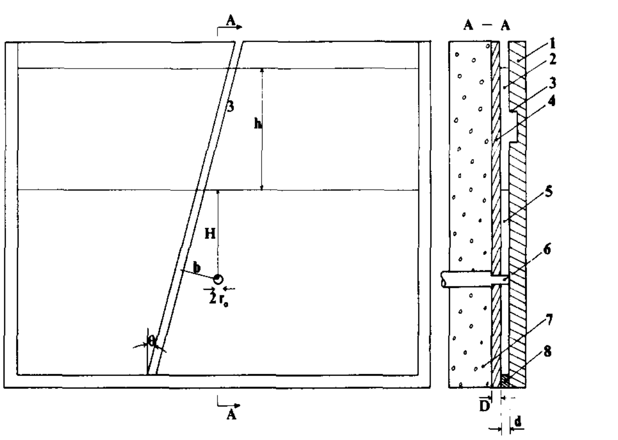

Figure 2 Drawing of the combined Hele-Shaw and heat conduction cell. 1, Plexiglas plate; 2, upper liquid; 3, possible hydraulic "short circuit"; 4, metal plate; 5, lower liquid; 6, heat source (connected to a soldering iron); 7, insulation; 8, seal

According to the theory in the previous section, valid for small Rayleigh numbers, one can expect that breakthrough is favored by great power, small density difference, small heat conductivity, and small thickness of the upper stratum. The same conclusions seem to be true for large Rayleigh numbers apart from the influence of the conductivity. Unfortunately a feasible experiment, i.e., the choice of adequate parameters, is restricted by practical limitations. Thus, the following conditions must be satisfied:

(1) The liquids must be immiscible.
(2) The density difference must be small.
(3) The liquids must not be harmful to Plexiglas.
(4) The volume expansion coefficient of the denser liquid must be equal to or greater than that of the less dense one.
(5) The liquids must have about the same viscosity.
(6) The heat source must be strong enough.
(7) Reynolds number based on the slot width must be small.
(8) The relative variation of the gap width must be small.
(9) The temperature must not be so high that the liquids or the Plexiglas are harmed.
(10) The liquids must be separated at room temperature.

The performed investigation showed that the greatest problem is to find two liquids simultaneously satisfying conditions 1, 2, 5, and 10. As an example it can be noted that if condition 2 is sacrificed conditions 1 and 5 can easily be satisfied, but this is obtained at the expense of conditions 7 and 10. It would have been desirable to have liquids with greater viscosity since condition 7 is not sufficiently well satisfied. It would have been possible to satisfy condition 7 by reducing the gap width, but this would have violated condition 8. Experiments with varying density difference would be of great interest. However, if the relative density difference is increased, condition 9 is violated and if it is decreased, condition 10 is violated.

Unfortunately the final choice of liquids and parameters is the least unfavorable alternative.

The components of the hybrid Hele-Shaw cell are the metal plate whose thickness is D and the gap width d.

A given pressure gradient ∇p induces a mean velocity $\mathbf{u}$ in the gap that reads

$$\mathbf{u} = -\frac{d^2}{12\mu}\nabla p \tag{42}$$

which implies that the pore velocity $\mathbf{v}$ is

$$\mathbf{v} = -\frac{nd^2}{12}\frac{1}{\mu}\nabla p \tag{43}$$

Thus, the equivalent permeability is

$$k \simeq \frac{d^2}{12} \tag{44}$$

Since the heat flow is assumed to be 2-D the variation of the temperature across the cell must be negligible. This is true as long as the Rayleigh number based on the gap width is small. In the present case r_0 and d are comparable and therefore the horizontal temperature variation across the cell can be neglected.

The choice of materials is described below.

$$\text{Silicon oil}\begin{cases}\rho_1 = 0.925\cdot 10^3 & \mathrm{kgm^{-3}}\\ c_1 = 2\cdot 10^3 & \mathrm{Jkg^{-1}K^{-1}}\\ \mu_1 = 4 & \mathrm{kg^{-1}s^{-1}}\\ \gamma_1 = 0.8\cdot 10^{-3} & \mathrm{K^{-1}}\end{cases} \tag{45}$$

$$\text{Maize oil}\begin{cases}\rho_2 = 0.921\cdot 10^3 & \mathrm{kg^{-3}}\\ c_2 = 1.7\cdot 10^3 & \mathrm{Jkg^{-1}K^{-1}}\\ \mu_2 = 6 & \mathrm{kg^{-1}s^{-1}}\\ \gamma_2 = 0.7\cdot 10^{-3} & \mathrm{K^{-1}}\end{cases} \tag{46}$$

Thus, the relative density difference is less than 0.5 percent. The back wall of the cell was made of two different materials, namely stainless steel and Plexiglas.

$$\text{Stainless steel}\quad \begin{array}{ll}\rho = 0.78\cdot 10^4 & \mathrm{kg^{-3}}\\ c = 0.45\cdot 10^3 & \mathrm{Jkg^{-1}K^{-1}}\\ \lambda = 0.6\cdot 10^2 & \mathrm{Wm^{-1}K^{-1}}\end{array} \tag{47}$$

$$\text{Plexiglas}\quad \begin{array}{ll}\rho = 1.18\cdot 10^3 & \mathrm{kg^{-3}}\\ c = 1.5\cdot 10^3 & \mathrm{Jkg^{-1}K^{-1}}\\ \lambda = 1.9 & \mathrm{Wm^{-1}K^{-1}}\end{array} \tag{48}$$

The thickness of the steel plate is

$$D = 10^{-2}\ \mathrm{m} \tag{49}$$

and the gap width is

$$d = 2\cdot 10^{-3}\ \mathrm{m} \tag{50}$$

Since the manufacturing accuracy of the steel plate and the Plexiglas plate is not better than 0.1 mm, the relative accuracy of the width of the gap is at most 5 percent. Hydraulic short circuits might be studied if a channel is milled in the Plexiglas plate. The distance between the possible short circuit and the heat source is denoted b and is shown in Figure 2.

The maximum power of the soldering iron is

$$Q_{\max} = 60\ \mathrm{W} \tag{51}$$

and the radius of the "source bar" is

$$r_0 = 2.5\cdot 10^{-3}\ \mathrm{m} \tag{52}$$

The possibility to simulate a hydraulic short circuit has not been used in the present study. This is equivalent with great distance between the source and the short circuit ($b \to \infty$ in Figure 2).

The figures above yield the following derived parameters:

$$\begin{array}{ll}\text{Steel plate} & n = 0.333\\ \quad + & k = 7\cdot 10^{-7}\ \mathrm{m}^2\\ \text{Plexiglas plate} & \mathrm{Ra}_{\max} \approx 3\cdot 10^{-6}\end{array} \tag{53}$$

The above figures correspond to a very permeable body composed of equal spherical balls.

If the steel plate is replaced by another Plexiglas plate, the Rayleigh number becomes $(\lambda_{\text{steel}}/\lambda_{\text{Plexiglas}})^2$ times, i.e., 1,000 times greater. In this case the heat transfer is dominated by the convection. Despite this the Rayleigh number is small.

$$\text{Plexiglas plate} + \text{Plexiglas plate}\qquad \mathrm{Ra}_{\max} \approx 3\cdot 10^{-3} \tag{54}$$

The experimental arrangement includes not only the test cell but also the power supply and a device for controlling the levels of the liquids in the cell. The power supply is a variable AC transformer and the power, which is proportional to the square of the voltage, is recorded

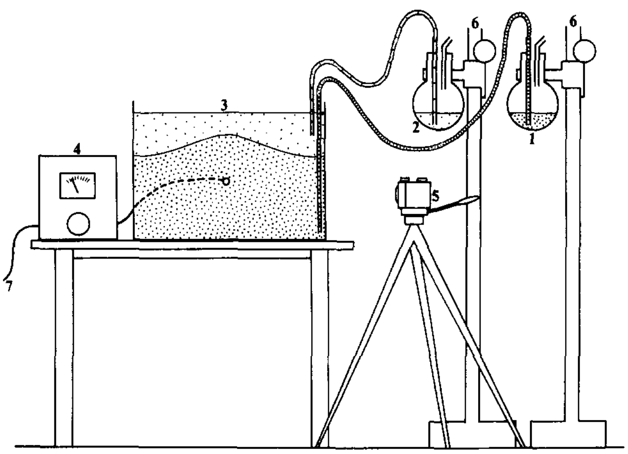

Figure 3 Experimental setup. 1, Silicon oil; 2, maize oil; 3, test cell; 4, variable transformer; 5, camera; 6, water level gauges; 7, power supply

with a voltmeter. The level of the liquids is controlled by two vessels fixed to two water level gauges. The arrangement is shown in Figure 3.

Results

As mentioned in the previous section, Q_c has been measured as a function of h and H. Q_c is given by the voltage over the soldering iron when the steady displacement of the interface touches the free surface, i.e., when $\delta_1(r = 0, t \to \infty) = h$. Due to the thermal inertia the rise time of the interface is long. An estimate of the time scale is given by the thermal diffusivity $\lambda/\rho c$ of steel and the dimension L of the cell. $\rho c L^2/\lambda \cong 3$ hours. As a matter of fact, steady state is reached after 7–8 hours. Unfortunately, it is easy to overestimate Q_c, and if a breakthrough has taken place, the heating must be terminated and the experiment restarted. Since the cooling is just as slow as the heating the experiments are very time consuming and the identification of Q_c is trying to the experimentalist.

The surface tension at the free surface forms an interesting complication. When the bump of the interface rises and approaches the free surface its curvature appears to be a smooth function, but if $Q < Q_c$ and if $\delta_1 \cong h$ the surface tension counteracts the breakthrough and the interface and the free surface are parallel and very close to each other over a distance that is about 10 times the gap width. Sudden breakthrough takes place and the upper liquid is separated into two parts. This is shown schematically in Figures 4 and 5. The transition from curve 2 to curve 3 in Figure 4 takes place within about 1 second. A similar phenomenon occurs when breakthrough has taken place. When the current is switched off and the interface gradually starts to return, the shape of the interface starts like curve 1 in Figure 5. The surface tension delays the development in that the transition from curve 2 to curve 3 in Figure 5 takes place within about 1 second.

The influence of the surface tension implies that Q_c is defined as the power that displaces the interface to a shape like curve 1 in Figure 4, i.e., before it is flattened like curve 2. With this definition, Q_c is independent of the complex phenomenon of the very breakthrough.

The experiments are evaluated from photographs, and Figure 6 shows a selection of them. The results of the experiments are shown in Figure 7. $\hat{Q}_c$ is defined in accordance with Equation 37, i.e., $\hat{Q}_c = Q_c(h/2r_0)/Q_c(h/2r_0 = 1)$. It is quite clear that $\hat{Q}_c$ is a linear function of $h/2r_0$ and also that $\hat{Q}_c$ is weakly dependent of $H/2r_0$. The broken line in Figure 7, which is the graph of Equation 38, shows that the deviation between a 2-D and a 3-D case is not too great. Two geometric configurations are especially interesting. If the heat source is located in the upper liquid ($H/2r_0 = -2$ in Figure 7) the lower liquid rises just like in the other cases and the interface breaks through, provided the power exceeds the critical value. The reason is that the temperature spreads equally in all directions, i.e., identically upward and downward. Thus, according to the simple theory Q_c is independent of $|H|$. This is of course also true if the heat source is located above the free surface even if this is of no interest in this context.

Conclusions

The theoretical analysis and the experiments described can now be used to calculate the critical power for a nuclear waste depository located in saline ground water below a stratum of fresh ground water. Let the salinity of the lower part of the ground water be C. The graph of the critical power $Q_c(h; C)$ is shown in Figure 8. The assumptions made in the previous sections are far-reaching and a simple situation like that shown in Figure 1b can hardly be found in nature. Nevertheless, it is worthwhile to study the consequences of Figure 8 in some practical case.

The analysis assumes, among other things, that the Rayleigh number is small and that the flow is quasi-steady.

For granite and water the following figures (in SI units) are typical: $\lambda = 3$, $n = 10^{-2}$, $\rho = 2.4 \cdot 10^3$, $c = 0.8 \cdot 10^3$, $k = 10^{-12}$, $\alpha = 10^{10}$, and $\mu = 10^{-3}$, $c_0 = 4 \cdot 10^3$, $\rho_0 = 10^3$, $\gamma_0 = 2 \cdot 10^{-4}$. If these figures are introduced in Equations 27 and 41 it is found

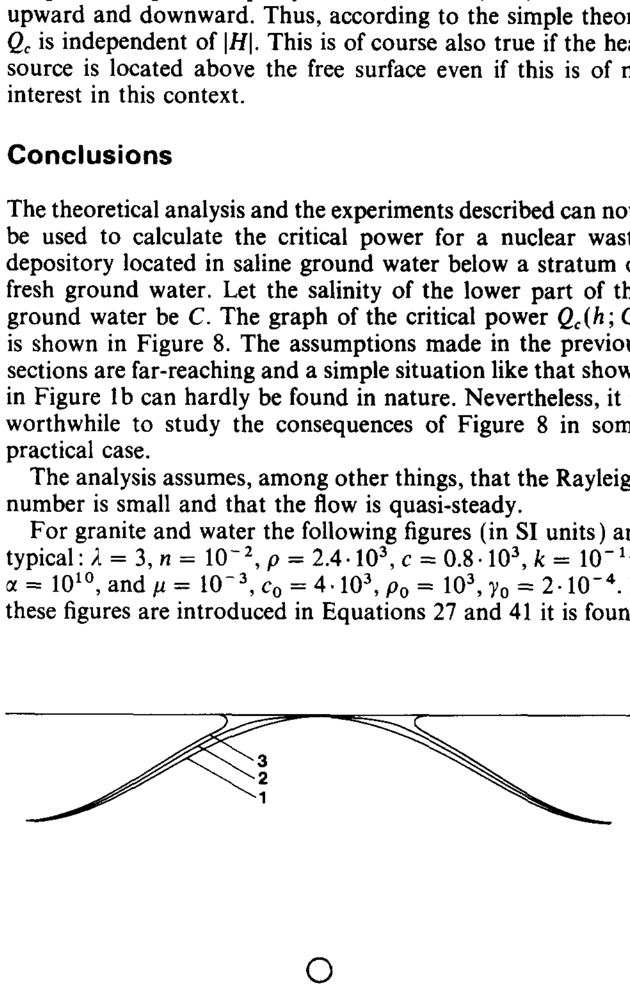

Figure 4 Sketch of the displacement of the interface when it is rising. 1, $Q = Q_c$; 2, $Q > Q_c$ (immediately before breakthrough); 3, $Q > Q_c$ (immediately after breakthrough)

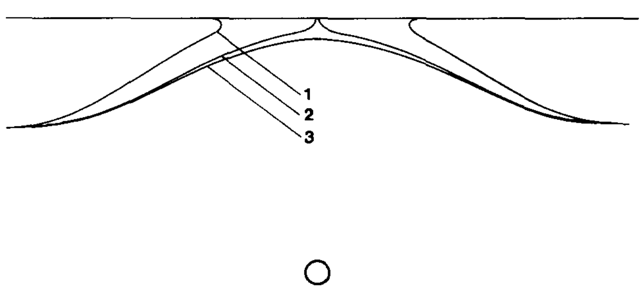

Figure 5 Sketch of the displacement of the interface when it is returning after breakthrough. $Q = 0$. 1, Maximum displacement; 2, immediately before closing the gap; 3, immediately after closing the gap

that

$$\mathrm{Ra} = Q \cdot 10^{-7} \tag{55}$$

$$\frac{\lambda n_0 \mu(\alpha + \beta)}{\rho c k_0} = 1.5 \cdot 10^{-11} \ll 1 \tag{56}$$

which indicates that the flow is quasi-steady and that the influence of convected heat can be neglected if the generated power is less than 1 MW.

Two sites in Sweden, namely Äspö and Finnsjön, have been investigated with respect to the groundwater situation. At Äspö, which is an island, the interface is a classical Ghyben-Herzberg interface and its maximum thickness is 40 m. The geohydraulic situation is not too complicated. At Finnsjön on the other hand, the situation is very complicated. The interface slopes as much as 12° in one direction whereas the free surface just slopes 0.1° but in the opposite direction. The thickness of the fresh water stratum varies between 100 and 300 m. The complicated situation is caused by zones of crushed rock whose permeability is 10^5 times or more that of the rock. It seems very likely that if a heat source were introduced at Finnsjön the Rayleigh number would not be small at all.

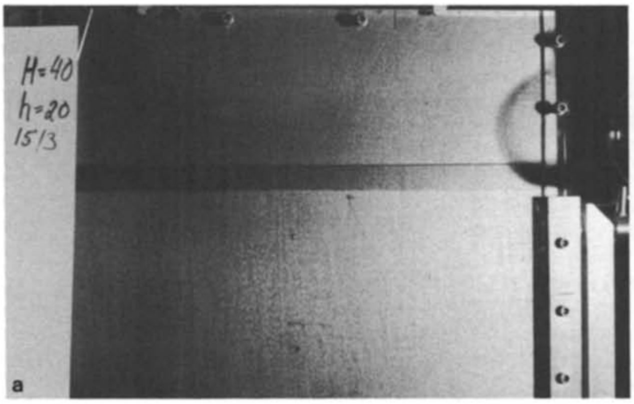

Figure 6 Photographs showing the displacement of the interface. a. Equilibrium. $Q = 0$.

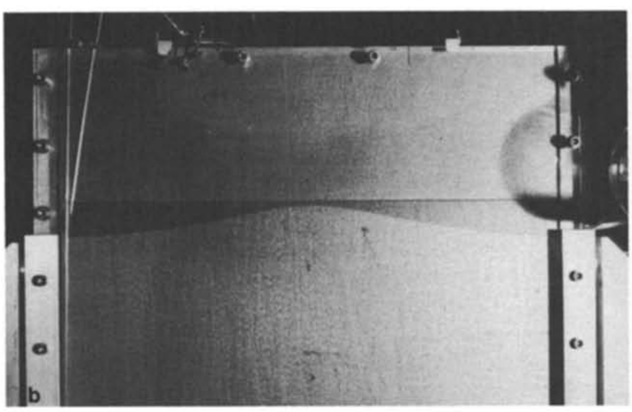

Figure 6(b) Nonequilibrium. $Q > Q_c$

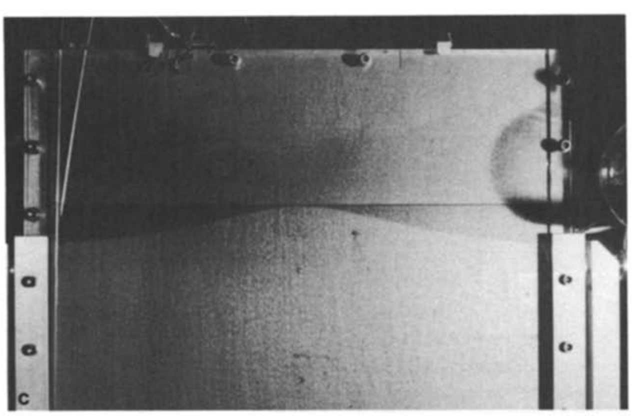

Figure 6(c) Equilibrium. $Q = Q_c$

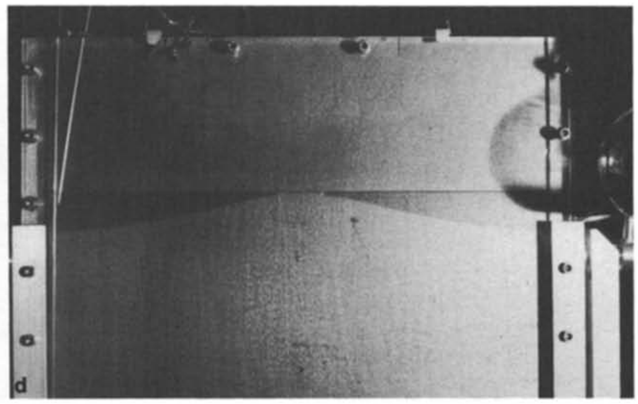

Figure 6(d) Nonequilibrium. $Q > Q_c$. Immediately after breakthrough

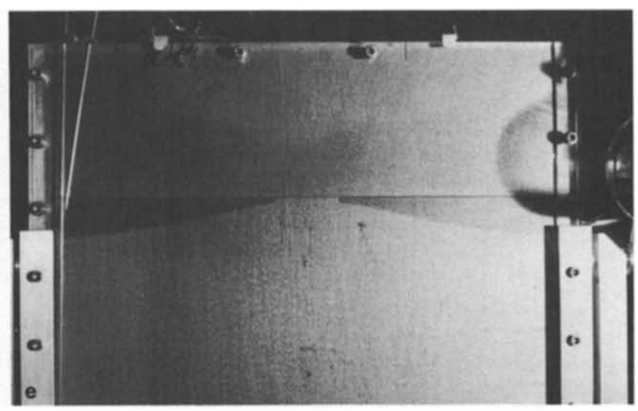

Figure 6(e) Nonequilibrium. $Q > Q_c$

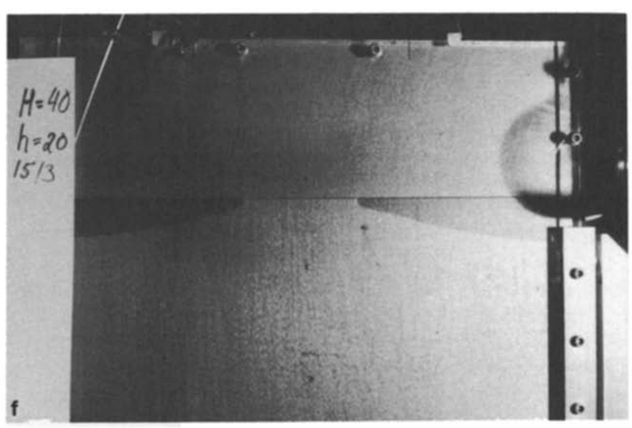

Figure 6(f) Equilibrium. $Q > Q_c$

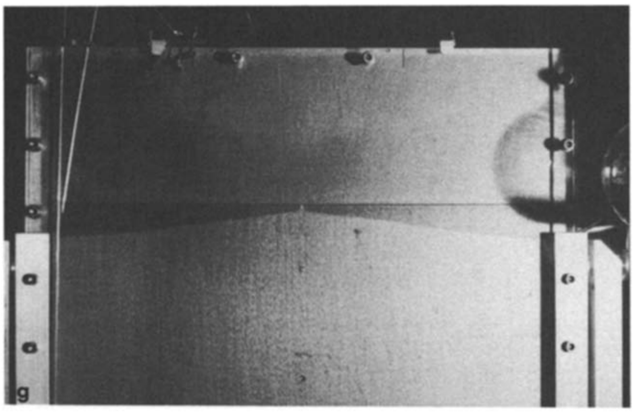

Figure 6(g) Nonequilibrium. $Q = 0$. Immediately before closing the gap

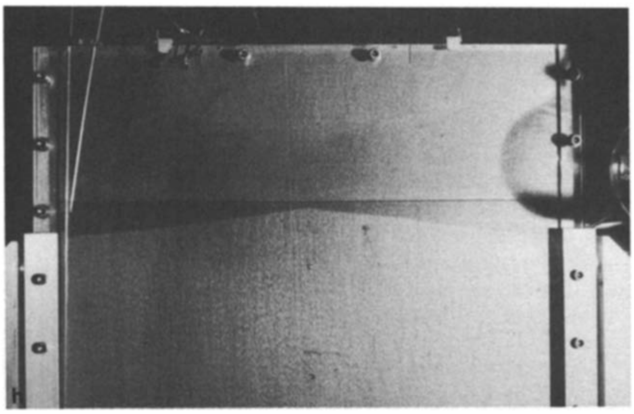

Figure 6(h) Nonequilibrium. $Q = 0$. Immediately after closing the gap

The salinity is about or less than 1 percent at both sites and Figure 8 indicates that for a depository at Finnsjön the critical power is about 200 kW whereas at Äspö it is not more than 35 kW. These figures are based on Equation 37, which in turn is based on the upconing approximation. As discussed previously, this approximation is verified for only one value of $(\rho_{10} - \rho_{20})/\rho_{10} = 0.00432$.

As mentioned above, the critical powers cannot be increased significantly if the depository is located at greater depth. It is clear from Equation 55 that these powers imply a small Rayleigh number. The question is now how these figures match the situation in a planned depository. In a depository the uranium is stored in capsules where every capsule produces 1 kW. The depository is planned to contain approximately 5,000 capsules. The produced power decreases with time and its half-life is 1,000 years. This means that the produced power is 5 MW and that steady state has been reached long before the power has decreased significantly. Thus, the produced power exceeds the critical power of Finnsjön and Äspö by a wide margin. As a matter of fact, the margin is so wide that breakthrough is expected even if Equation 37 overestimates the influence of the relative density difference.

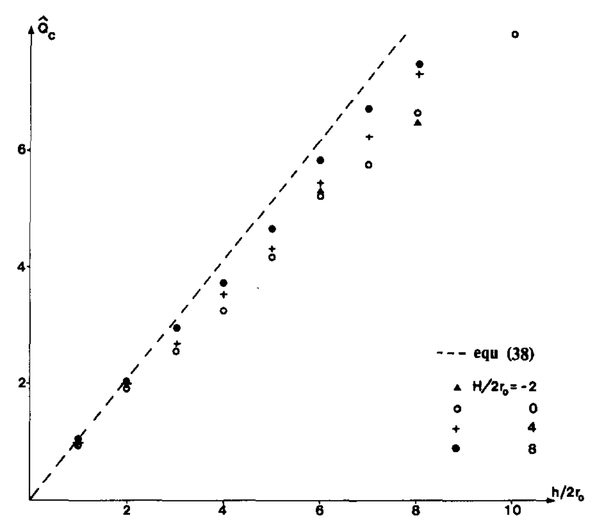

Figure 7 The dimensionless critical power as a function of the dimensionless thickness of the upper stratum and with the dimensionless distance between the heat source and the interface as a parameter

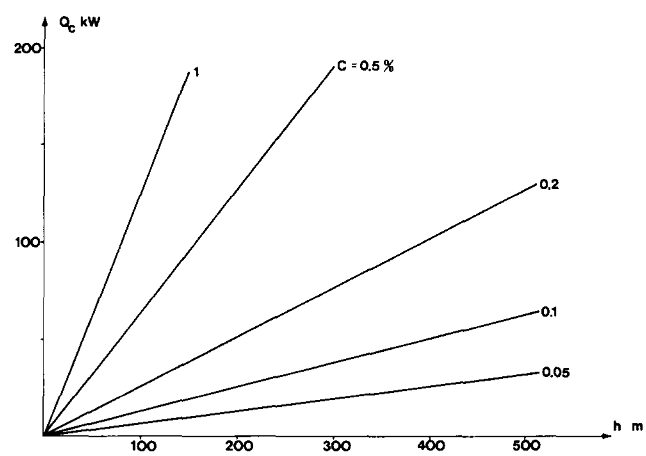

Figure 8 The critical power as a function of the thickness of the fresh water stratum and with the salinity as a parameter

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